Chaotic Josephson oscillations of exciton-polaritons and their applications

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We consider Josephson oscillations between two coupled, quasiresonantly pumped exciton-polariton (polariton) modes. We analyze the regions of stability and periodic oscillations of such a system and show that the spin degree of freedom allows a chaotic behavior. We show that two spatially separated polariton Josephson junctions can be synchronized and used for data encryption and transfer. Due to the strong nonlinearity of polariton-polariton interactions, chaos communication at rates up to 50 Gb/s is possible, which is at least ten times faster than the results obtained for other systems.

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I. INTRODUCTION

Chaotic behavior of perfectly deterministic systems has been fascinating scientists since its discovery by Poincaré[1] and its popularization by Lorenz[2] and Mandelbrot.[3] Chaotic behavior has been observed in nature as well as in laboratory in a variety of systems including mechanical, chemical, biological, electrical, and optical ones. A necessary condition for a system to be chaotic is nonlinearity. Moreover, the complexity of the system should be sufficient in order to allow the existence of the chaotic attractor in the phase space. After the seminal paper of Pecora and Carroll,[4] the question of synchronization of chaotic systems has become one of the central ones in the field of nonlinear dynamics. The possibility to apply chaos synchronization to communications immediately became apparent. Several experimental implementations of this concept have been demonstrated,[5] including ones based on a semiconductor laser.[6] Chaos communication at a distance of 120 km using commercial fiber-optic links has been demonstrated in 2006.[7] Although the questions of security of chaotic communications are not always analyzed thoroughly for the proposed systems, a certain general analysis for the field has been carried out recently.[8]

In this paper we present a system capable of demonstrating chaotic behavior and synchronization. This system is a Josephson junction between spinor cavity exciton-polariton macro-occupied modes.[9] Chaotic Josephson oscillations and their synchronization have been studied before in arrays of superconductors with Josephson junctions but with a different nonlinear term with respect to the case we consider.[10,11] Cavity exciton-polaritons (polaritons) are the eigenmodes resulting from the strong coupling[12] between quantum-well excitons and cavity photons in microcavities. The polaritons are now a subject of intense fundamental and applied studies[13] because of their peculiar properties. Indeed, the excitonic component of the polaritons makes efficient their interactions between each other and with the environment, while the photonic component provides a very light effective mass (10^{-5} of the electron mass), extended coherence, and the possibility to optically excite polariton states with high efficiency. These properties, together with the bosonic behavior in the low-density limit,[14] have recently allowed to demonstrate polariton Bose condensation at low[15] and high[16] temperatures, as well as other interesting effects, including superfluidity[17] and vortex formation.[18]

The polarization or spin properties of the polaritons have also been investigated thoroughly (see Ref. 13 for a review). Here we will use some of these properties, notably that the polaritons can have two spin projections on the growth axis (+1 and −1) and are therefore described by two polarization components. The polariton-polariton interaction is spin anisotropic: strongly repulsive for excitons with the same spin projection on the growth axis and weakly attractive for excitons with opposite projections.[19,20] Under quasiresonant pumping, a spinless polariton mode can be considered as a nonlinear oscillator demonstrating bistable behavior.[21,22] If the polariton spin degree of freedom is taken into account, a polariton state behaves as two coupled nonlinear oscillators, showing multistable behavior.[23] On the other hand, another polariton system, which behaves as two coupled nonlinear oscillators as well, consists of two spatially separated polariton states with Josephson coupling between them (but without polarization). Such system has been recently considered by Sarchi *et al.*[24]

However, the two systems described above are not complex enough in order to allow a chaotic behavior. The complexity can be further increased by considering the polariton Josephson junction taking into account the polarization degree of freedom. This system of four coupled nonlinear polariton oscillators has been recently analyzed,[9] showing a rich phenomenology. The coupling between two spin components without spatial separation provided by the natural splitting of the ground state due to the anisotropy of the cavities[25] was called intrinsic Josephson effect because it does not require any engineering of the spatial polariton confinement. In the nonlinear regime, the coupled macro-occupied polariton modes have been predicted to exhibit macroscopic quantum self-trapping leading to a spatial separation of the spin components.[9]

In the present work we consider two traps with two polarization components in each trap with both extrinsic and intrinsic coupling mechanisms present. These couplings bring in the complexity sufficient to observe chaotic behavior. We first demonstrate the chaotic behavior of a single system of two traps, analyze its properties, and then demonstrate chaos synchronization of two such systems and its application to chaotic communications. We also consider the case of polarization-independent Josephson oscillations which involve three spatially separated subsystems.

II. CHAOTIC JOSEPHSON OSCILLATIONS

We consider two potential traps of 3 μm separated by a potential barrier. Such a potential profile can be realized by applying a stress on the microcavity surface[26] or by patterning two micropillars[27] close to each other. In the single-mode mean-field approximation one can write the equations for the amplitudes of the polariton fields $\psi_{j\sigma}$ (σ is the polarization component and j is the trap number).

$$i\hbar\frac{\partial\psi_{1,2\pm}}{\partial t} = -i\frac{\hbar\psi_{1,2\pm}}{\tau} + \alpha_1|\psi_{1,2\pm}|^2\psi_{1,2\pm} + \alpha_2|\psi_{1,2\mp}|^2\psi_{1,2\pm} - J\psi_{2,1\pm} - W\psi_{1,2\mp} + P_{1,2\pm}e^{-i\omega t}, \quad (1)$$

where $\alpha_1 = 6xE_b a_B^2/S$ is the polariton-polariton interaction constant in the triplet configuration with x being the exciton fraction of the polariton mode and S being the area of the traps,[19] $\alpha_2 \cong -0.1\alpha_1$ is the polariton-polariton interaction in the singlet configuration,[20] J is the coupling strength between two spatially separated modes (extrinsic Josephson coupling), which can be tuned by adjusting the distance between the pillars. W is the coupling strength between two circularly polarized modes in the same trap (intrinsic Josephson coupling), which, for instance, can be tuned by growing elliptic pillars showing linearly polarized eigenmodes split in energy, or applying directional stress. $P_{j\sigma}$ is the pumping intensity of the external laser, where only P_{1+} is different from zero. $\hbar\omega$ is the laser detuning, or the energy difference between the pumping energy of the laser and the energy of the bare polariton mode $\hbar\omega_p$ (without Josephson coupling and interactions), which we take as the zero reference. Finally, $\tau/2$ is the polariton lifetime.

The single-mode approximation we use neglects the states that are not occupied macroscopically and is valid until these neglected states are stable against polariton scattering toward them. In a two-dimensional system one has to check the stability against parametric scattering toward a pair of states of the continuum. In a confined system it is sufficient[24] to verify that the energy difference between the first excited state and the ground state is much larger than the detuning of the pump laser and than the blueshift. For a square trap we consider, this energy difference can be estimated as $3\pi^2\hbar^2/2mL^2 \approx 6$ meV (depending on the exciton-photon detuning), which is much larger than the pump detuning values used below. However, special attention should be paid in experimental implementations to the depth of the wells and to their size, in order to avoid the excitation of other modes.

In the calculations we use the parameters of a typical cavity with GaAs quantum wells since such cavities presently show the longest lifetimes (we take $\tau = 20$ ps) and can

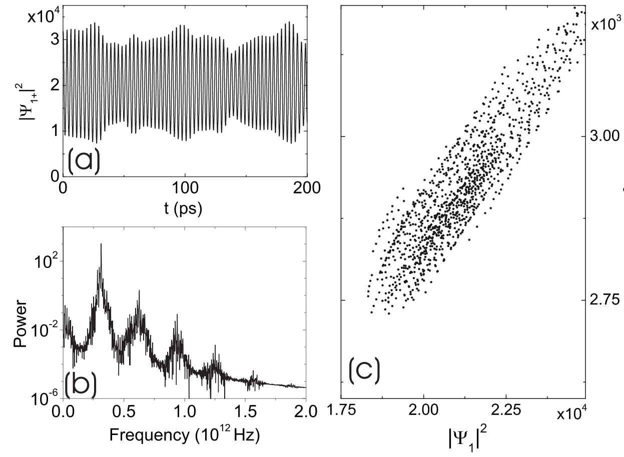

FIG. 1. (a) Spin-up component from the first trap (the one which is pumped) as a function of time; (b) Fourier transform of the spin-up component showing chaotic spectrum capable of masking useful signal at least up to 10^{12} Hz; and (c) Poincaré cross section of the phase space.

be easily patterned. This choice of material however implies low-temperature operation. Future practical implementations could be based on large band-gap semiconductors, such as GaN, for which room-temperature polariton Bose Einstein condensation has been reported.[16] We take $W = 150$ μeV, in the range of values measured experimentally, and $J = 90$ μeV. Both should be close to each other and to $\hbar/\tau$, in order to observe the chaotic behavior.

The results of numeric simulations performed using the system of Eq. (1) are shown in Fig. 1. Panel (a) shows the time dependence of $|\psi_{1+}|^2$ for a set of parameters providing chaotic behavior. The great advantage of the polariton system is that the light intensity emitted by the system is directly proportional to the polariton density and therefore shows the same time dependence. Figure 1(b) shows the frequency spectrum obtained from the Fourier transform of $\psi_{1+}(t)$ and the Fig. 1(c) the Poincaré cross section (a cut of the multidimensional phase space by an arbitrary chosen plane). These two figures demonstrate typical signatures of the chaotic behavior: a continuous Fourier spectrum with a random peak structure and a fractal structure of the Poincaré cross section correspondingly (for a closed trajectory, the image would be a finite set of points).

A nonlinear system can be efficiently analyzed using the conditional Lyapunov exponents, that is, the logarithms of the eigenvalues λ_i of a function of the Jacobian matrix of the system

$$\lim_{t\to\infty}[J^*(t)\cdot J(t)]^{1/2t}$$

calculated along its trajectory (see Ref. 28 for review and numerical method description).

The maximal Lyapunov exponent characterizes the rate of separation in time of the trajectories which are initially infinitesimally close,

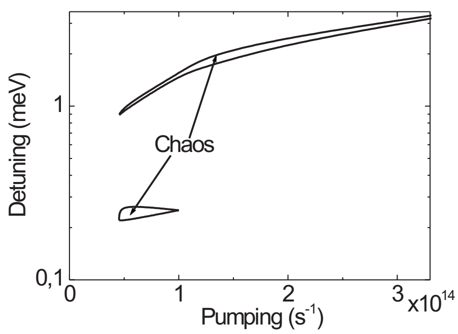

FIG. 2. Phase diagram of the double-trap system with two polarizations. Solid lines mark the boundaries of the regions with chaotic behavior.

$$\lambda_{\max} = \lim_{t\to\infty} \frac{1}{t}\ln\frac{|\delta\Psi(t)|}{|\delta\Psi(0)|},$$

with $\delta\Psi(t)=\sqrt{\Sigma_{j=1,2;\ \sigma=\pm}(\delta\psi_{j\sigma})^2}$. Chaotic regime is thus characterized by the presence of positive Lyapunov exponents, which indicate the divergency of any small initial perturbation. Qualitatively, the chaotic regime is obtained when the bistability transition of the mode pumped directly (ψ_{1+} in our case) is constantly invoked by the changes of its population due to the Josephson oscillations of both types.

To carry out quantitative analysis, we have performed simulations at different pumpings P_{1+} and detunings ω (which are the most easily variable parameters in experiments since they are determined only by the external laser). Figure 2 summarizes the results of these simulations showing the phase diagram of the system in the detuning-pumping intensity coordinates, the other parameters of the system being kept constant. The boundaries of the chaotic regions are shown by black solid lines. One can see that almost for any detuning under certain pumping the system should exhibit chaotic oscillations. Outside the chaotic regions, the oscillations are periodic or absent, as already predicted for a simpler system.[24] Another strong specificity of this system is the very low injected power[23] which is required to achieve a nonlinear and even chaotic behavior of the system. This is due partly to the small size of the system considered, to the long particle lifetime considered, but also to the intrinsic properties of polaritons, namely, their capacity to be resonantly excited because of their photonic component, and to their strong nonlinear response due to their excitonic component which brings a strong polariton-polariton interaction.

III. CHAOS SYNCHRONIZATION

Synchronization of chaotic systems is a necessary condition for the implementation of chaotic cryptography. Several types of synchronization are possible. We use the so-called Pecora-Carroll method.[4] We divide our system into two subsystems, one of them being intrinsically chaotic, and the other intrinsically stable against small perturbations. These two conditions are checked calculating the conditional Lyapunov exponents along the trajectory in the chaotic regime. This calculation shows that for the chosen parameters (pumping $1\times10^{14}\ \mathrm{s}^{-1}$, detuning -1.5 meV, appearing to be optimal values) the system can be divided into two subsystems, one of them associated with the first trap, and the other with the second trap. The Lyapunov exponents are positive for the wave functions of the first subsystem and negative for the wave functions of the second subsystem. Therefore the synchronization between two systems should be performed by connecting the output of the first trap of the master system (both polarization components) with the only trap of the slave system (with parameters corresponding to the second trap of the master system), as shown by the blue arrows (marked synchronization) in Fig. 3(a). We describe this numerically by adding a term proportional to the wave function of the driving system $V\psi_{1\pm}(t)$ to the equations describing the slave system $\psi'_{2\pm}(t')$.

$$i\hbar\frac{\partial\psi'_{2\pm}}{\partial t'} = -i\frac{\hbar\psi'_{2\pm}}{\tau} + \alpha_1|\psi'_{2\pm}|^2\psi'_{2\pm} + \alpha_2|\psi'_{2\mp}|^2\psi'_{2\pm} - V\psi_{1\pm}(t) - W\psi'_{2\mp}. \tag{2}$$

The coupling V represents the only source of pumping for the slave system. A certain time delay, which should be taken into account when decoding a useful signal, is present between the master and the slave systems.

IV. CHAOTIC COMMUNICATION

To transmit information hidden in the chaotic oscillations we introduce a communication channel to the system shown by the red lines (marked communication) in Fig. 3(a). A useful digital signal with a repetition rate of 50 GHz is added to the output of the second polarization component of the master system in point 1 and transmitted to the receiver 2, where the useful signal is reconstructed by the comparison with the output of the synchronized slave system. The corresponding Fourier spectrum of the chaotic output with the added useful signal is shown in Fig. 3(b), where the arrow indicates the frequency position of the masked useful signal. Obviously, the oscillations invisible in the Fourier spectrum are completely masked in the time domain as well (not shown). The signal s itself is shown in (d). The subtraction of the chaotic output of the synchronized slave system from the received signal from the master system allows recovering of the transmitted signal [Fig. 3(e)].

Figure 3(c) shows the relative synchronization error between two corresponding wave-function components $(|\psi'_{2+}|^2 - |\psi_{2+}|^2)/(|\psi'_{2+}|^2+|\psi_{2+}|^2)$ from the driving and the slave systems with a mismatch of 0.5% of the coupling between polarizations within the same well W. Although there are several peaks rising up, the synchronization is always recovered on the long term, allowing a continuous signal transmission for communication.

The chaotic oscillations for the given parameters allow signal frequencies up to some THz for extremely well-synchronized systems. But the recovering of the transmitted signals is restricted by the frequency spectrum of the synchronization error caused by the parameter mismatch of the coupling constants and by the noise, which are always present in experimental setups.

One of the most important aspects of a cryptographic system is its security. A set of criteria for estimation of the

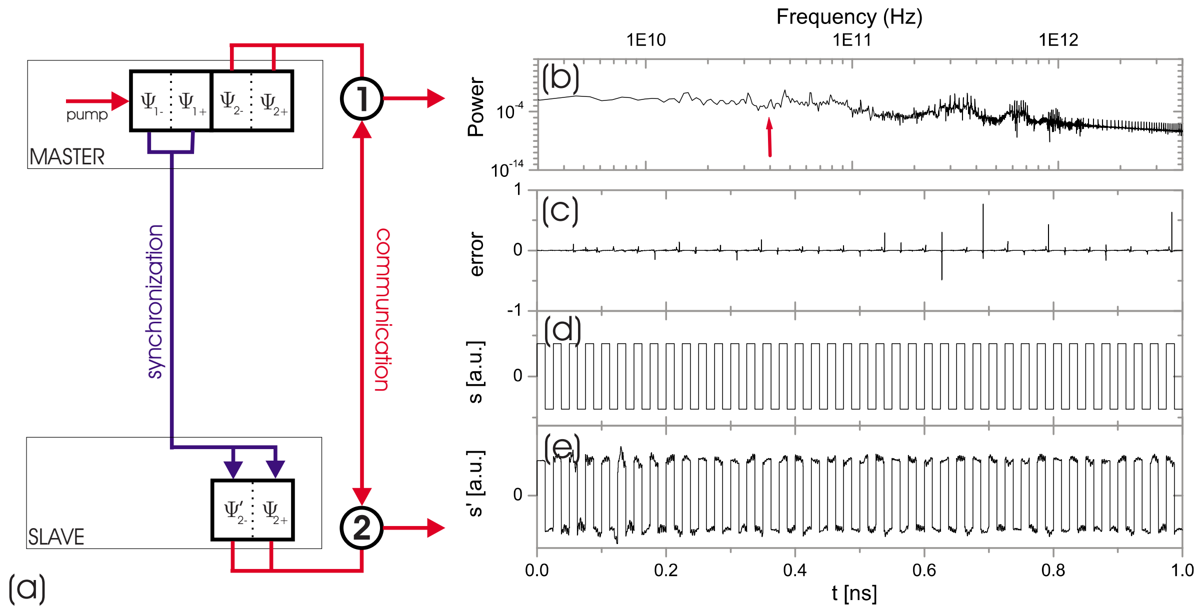

FIG. 3. (Color online) Synchronization of the chaotic systems and chaotic communication: (a) schematic system with a separated synchronization channel (blue) and a communication channel (red); (b) Fourier spectrum of the chaotic behavior of the second polarization component of the master system modulated with a signal (red arrow); (c) synchronization error without signal versus time; (d) transmitted signal s versus time, and (e) recovered signal s'.

security of chaotic cryptographic systems has been proposed recently.[8] These criteria include, for example, resistance to message signal extraction attacks: the useful signal should not be visible in the Fourier spectra. Another criterion is the robustness against noise in the communication channels. We have checked that such criteria are well verified for our system.

The experimental implementation of this polarization-dependent communication scheme might seem challenging due to technical restrictions, such as that the polarization is not maintained during the transmission through an optical fiber. Therefore, we have tested a similar system without the polarization degree of freedom. However, to have sufficient complexity, it is necessary to have at least three traps, with two Josephson couplings between them. This can be obtained by arranging the traps in a line and pumping the central one. In this configuration one can observe chaotic oscillations and their synchronization with the slave system consisting of a single trap. However, since the coupling between the two circular polarizations within a trap is always present because of the quantum-well anisotropy, this model, although working, is not an adequate representation of the real system. We therefore recommend the first scheme (two traps, two polarizations) for realizing the chaotic communication systems.

Using separate synchronization and communication channels opens the possibility of bidirectional communication while the synchronization is provided by a unidirectional coupling of the master and slave system. Within such a scheme, one can even implement chaotic communication networks. A scheme of a chaotic communication network contains a server (the master system), driving several slave systems. Once all slave systems are synchronized with the master, one can send and receive messages from one slave to another. The knowledge of all "flight" times of the signal between the slave systems is a necessary requirement to reconstruct the information.

V. CONCLUSIONS

In conclusion, we have studied chaotic Josephson oscillations between two macro-occupied polariton modes under quasiresonant pumping, taking into account the polarization degree of freedom. This unique system has the advantages to directly deal with optical useful signal but simultaneously to show very strong nonlinearities due to the polariton-polariton interaction. As a result, it shows a chaotic behavior for very low external pumping intensities with characteristic power spectrum expanding in a very high-frequency range. Chaos synchronization is possible for such systems, allowing the transmission of useful signals with chaotic masking at rates up to 50 Gb/s, which is an important advantage with respect to other optical chaotic communication systems.

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